NEW PUZZLES ON KNIGHT-GRAPHABLE WORDS

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In a previous article [1] we examined some properties of knight-graphable words. A word is *knight-graphable* (NG) if the distinct letters of the alphabet used in the word can be placed on separate squares of an arbitrarily large chessboard such that a chess knight can spell out the word by starting on the square labelled with the word's first letter and making a series of knight's moves to the successive letters in the word. Note that the word is allowed to contain repeated runs of the same letter; the knight simply stays put for a "move" from a letter to itself. For example, UNDISGUISED is an NG word, as shown by this chessboard labelling:

		U		
I				G
		S	N	
	D			
			E	

In this article we will discuss two new challenges involving knight-graphable words.

Puzzle #1: Maximum Word-Capturing Alphabet Layout

This puzzle is easy to state but not so simple to solve. Place each of the 26 letters of the alphabet on a different square of an infinite chessboard in any arrangement you wish. Now count up how many words can be spelled by a square-hopping chess knight, where for each word you can start on any letter and use any sequence of knight moves to spell the word. Which arrangement of the alphabet maximizes the number of words that can be spelled by a knight (i.e., maximizes the number of NG words contained in the grid)?

To make this puzzle precise we have to decide on a list of valid words. We decided to use the second edition of the North American Scrabble *Tournament and Club Word List*, a word list known as TWL06, as our arbiter. How many of the 178,691 words in this list (of which only 101,702 are knight-graphable) can we spell from a single arrangement of letters using knight's moves?

This is one of those puzzles where computer assistance helps greatly, but even with a computer this is a difficult problem. The number of possible arrangements, even if we impose some restrictions such as requiring the arrangement to be completely connected with no isolated letters, is much too large to exhaustively check every one. So some kind of heuristic must be employed.

We chose to use a "hill-climbing" strategy. Generally speaking, a hill-climbing algorithm starts from some configuration (say, a completely random assignment of letters to squares of the chessboard) and continually tries to improve it via small "tweaks". A small change to the arrangement is examined to see if it is better, and if so then this new arrangement is accepted and the process is repeated. In the application at hand we defined a tweak to be randomly choosing an integer *n* between 1 and 6 and then doing this *n* times: pick a pair of squares on the chessboard at random (with at least one of the two squares containing a letter) and swap the contents of those squares. A hill-climbing strategy must also contain a "restart clause": if we've tried to tweak the current solution M times in a row unsuccessfully, then we give up and start over from another random arrangement. We used M = 100,000.

We ran this algorithm for about eight hours and noticed that many of the high-scoring grids contained a construction like this:

	c		c	
c				c
v		E		v
c				c
	c		c	
	v		v	

Where E is the letter E, the four squares marked "v" contain the other vowels A,I,O,U in some order, and the common consonants T,N,R,S,D occupy five of the "c" squares, which puts them one step away from the E, providing (among other things) the useful prefix RE- and the suffixes -EN, -ER, -ES, and -ED. So we changed our program to always include a pattern like this in the starting arrangement and let it run for about 12 hours. The highest-scoring grid that we found is this one,

	H		P		N		X	
J		W	G	M	Y	V		
		O		E		I		
		R		L		D		Z
	C		S		T		F	
			A		U			
		K						
		Q		B				

which can be used to spell 1478 TWL06 words (14.5% of all the NG words available):

aa aal aalii aargh aarrgh aarrghh ab aba abaca aback abate abated abater abates abating abba abuse abused abuser abuses acock al ala alack alar alate alated alates alif aline alined aliner alines alining alit aliterate aliterates all alliterate alliterated alliterates alliterating allocate allocated allocates allocating allude alluded alludes ally allyl alula alular ar are arene arenes arenite arenites ares arete aretes argling arrack arracks at atabal ate ates att attack attacks attar ba baa baal baalim baba babu babul babus bacca baccara baccarat baccate baccated back backs backset bal balata baling ball balling bally bar bare bared bareness barenesses barer bares barrack barracks barrater barre barred barren barrener barrenness barrennesses barres barret barrette barrettes bat bate bated bates bating batt batted batten battened battener battening batter battered batterer battiness battinesses batting bub bubal bubaline bubba bubu bubus bud budded budder buddy buff buffi buffing bull bulla bullate bulling bullock bullocks bully bus bused buses busk busks buss bussed busses cab cabal cabala caballing cabaret cabbala caca calif calix call calla callaloo callaloos calling callose calloses callus callused calluses calo calos calyx car caracal caracara carack caracks caracol caracoling caracolling carat carate carates care cared careen careened careener careening career careered careerer carer cares caress caressed caresser caresses caret carr carrack carracks cat catalo catalos cate cater catered caterer cateress cateresses cates cattalo cattalos catted cattily cattiness cattinesses catting coca coccal cock cocks coco cocos coho cohos col cola colin collar collared collaret collate collated collateral collateralizing collaterally collates collating collocate collocated collocates collocating collop collude colluded colluder colludes colly colocate colocated colocates colocating colossus colossuses coly coming comingling comix commingling commit committal committed committee committees committing commix commixing comose coo coocoo cool cooling coolly cooly coop cooped cooper cooperate cooperated cooperates cooperating coopered coos cop cope coped copen coper copes copped copper coppered copy copydesk copydesks cos coses coset coss cosset cosseted cosseting de dee deed deeded deedy deep deepen deepened deepener deepening deeper deer dees deet den dene denes deni denim denned denning depose deposed deposer deposes derat derate derated derates derating deratted deratting dere desk desks desolate desolated desolateness desolatenesses desolater desolates desolating desolatingly deter deterred deterrer dev dew dewed dub dud duddy dude duded dudeen dudes duff dufus dufuses dull dulling dully duly dusk dusks ed eddy en eng epee epees epergne epergnes epopee epopees epos eposes er era ere erg err errata erred es eses ess esses et eta eve even evened evener evening evenness evennesses ever evergreen eves ewe ewer ewes fifing fil fila filar filaree filarees filing fill filling fillo fillos filly filo filos filose fin fine fined fineness finenesses finer fines finesse finessed finesses fining finite finiteness finitenesses finites finned finning fit fitted fitter fitting fittingly fittingness fittingnesses fix fixing fixit fiz fizz fizzing fub fubar fud fuddy full fulling fully fuse fused fusee fusees fuses fuss fussed fusser fusses glare glared glares gliff glim gliming glitter glitterati glittered glom glomming gloom gloomily gloominess gloominesses glooming glop glopped gloppy gloss glossed glosser glosses graal grab grat grate grated grater grates gratin gratine gratinee gratineed gratinees grating gratingly gree greed greedy green greened greener greening greenness greennesses grees greet greeted greeter greeting grew ho hock hocks holily holiness holinesses holing holla hollo holloo holloos hollos holly holy homily homines hominess hominesses homing hominine hominizing hommock hommocks hommos hommoses homo homopolar homos hooly hoop hooped hooper hoopoo hoopoos hop hope hoped hoper hopes hopped hopper hoppy hos hose hosed hosen hoser hoses if iff iffiness iffinesses ill illite illiterate illiterateness illiterates illites illude illuded illudes illy imine imines imitate imitated imitates imitating immingling immix immixing immolate immolated immolates immolating in ingrate ingrates ingress ingresses inn inned inner inning it iterate iterated iterates iterating itinerate itinerated itinerates itinerating jo jock jocks jocose jocoseness jocosenesses jollily jolliness jollinesses jolly joss josses kbar la lab labara lac lack lacks lall lalling lar laree larees lares lat late lated lateen lateener laten latened lateness latenesses latening later lateral lateraling lateralizing lateralling laterally lati latilla latinizing latte latten latter lattes lattin li lilac lilo lilos lily liminess liminesses liming limit limited limiter limites limiting limitingly limo limos lin line lined linen liner lines ling linin lining linn linnet lit lite liteness litenesses liter literal literalizing literally literate literateness literatenesses literates literati literatim litten litter littered litterer lo loca local localite localites localizing locally locate located locater locates locating lock locks lockset loco locos loll lolling lollingly lollop lolloped lollopy lolly lollypop loo loom looming loop looped looper loopy loos loose loosed loosen loosened loosener looseness loosenesses loosening looser looses lop lope loped loper lopes lopped lopper loppered loppy lose loser loses loss losses lude ludes luff luffing lull lulling lulu lulus lusus lususes mi miff miffiness miffinesses miffing mil militate militated militates militating mill milliliter millimolar milline milliner millines milling milo milos mim miming mine mined miner mineral mineralizing mines mingling mini minilab minim minimill minimizing mining mite miter mitered miterer mites mitt mitten mittened mix mixing mm mo moc mock mocks mojo mojos mol mola molal molar moline moll mollusk mollusks molly moly mom momi moo mool moola moos moose mop mope moped moper mopes mopped mopper moppet mopy mos mosk mosks moss mossed mosser mosses mosso ne neddy nee need needed needer needy neep nene nenes nepeta ness nesses net nett netted netter netting neve never neves new newer nil nill nilling nim nimming nine nines nineteen nit nite niter nites nix nixing nixy oca oh oho olla om omit omitted omitter omitting ooh oolite oolites op ope oped open opened opener opening openness opennesses opera operate operated operates operating operetta opes oppose opposed opposer opposes os ose oses pe ped pedes pee peed peen peened peening peep peeped peeper peer peered peeress peeresses pees peeve peeved peeves peewee peewees pen penes penne penned penner penni pennine pennines penning pep pepo pepos pepped pepper peppered pepperer peppy per pere peres pes peseta peso pesos pet petal petaline peter petered petit petite petiteness petitenesses petites petted petter petti pettily pettiness pettinesses petting pew pewee pewees pock pocks poco pocosen poh pol polar poling polite politeness politenesses politer politesse politesses poll pollack pollacks polling pollinizing pollock pollocks polo polos poly polyp polyped pom pomo pomos poo pooh pool pooling poop pooped poos pop pope popes popped popper poppet poppy pose posed poser poses posse posses possess possessed possesses posset pyx qabala qat rabat rack racks ralline rally rare rared rareness rarenesses rarer rares rat ratal ratatat rate rated rater rates ratine ratines rating ratite ratites ratted ratteen ratten rattened rattener rattening ratter ratting re red redd redded redden reddened reddening redder rede reded redes redub ree reed reeded reedy rees reeve reeved reeves renew renewed renewer renin rennet rennin rep repo repoll repolling repos repose reposed reposer reposes repossess repossessed repossesses repp repped rerack reracks res resee reseed reseeded reseen resees reset resetter resetting resew resewed resoling ret retack retacks retally rete retene retenes retiling retiming retine retinene retinenes retines retinite retinites retted retting rev revere revered reverer reveres revet revetted revetting revved rewed rewedded rewet rewetted rewetting seder see seed seeded seeder seedy seen seep seeped seepy seer seeress seeresses sees sen sene seniti sennet sennit ser sera serac seral sere sered serene sereneness serenenesses serener serenes serer seres serrate serrated serrates serrating set seta setal sett settee settees setter setting seven sever several severally severe severed severeness severenesses severer sew sewed sewer sewered so soca sock socks sol sola solar solate solated solates solating soli soling solo solos solus som somital somite somites sop sopped soppy sos sub suba subalar subulate sudd sudden suddenness suddennesses suffix suffixing suffuse suffused suffuses sully sulu sulus suss sussed susses ta tab tabaret tabu tabular tabulate tabulated tabulates tabulating tabuli tabus tack tacks taco tacos tala talar tali tall tallit tallitim tallol tally talus taluses tar tare tared tares tarre tarred tarres tat tatar tate tater tates tatted tatter tattered tattily tattiness tattinesses tatting ted tedded tedder teddered teddy tee teed teen teener teepee teepees tees teeter teetered ten tenet tenner tepee tepees terete terra terreen terrene terrenes terret tessera tet tetter tew tewed ti tiff tiffin tiffined tiffing tiffining til tiling till tilling tillite tillites timing timolol tin tine tined tines ting tingling tinglingly tingly tinily tininess tininesses tining tinned tinner tinnily tinniness tinninesses tinning tit titer titi titillate titillated titillates titillating titillatingly titter tittered titterer udder ulu ululate ululated ululates ululating ulus us use used user uses vedette vedettes vee veep veepee veepees veer veered vees veneer veneered veneerer venene venenes venerate venerated venerates venerating venin venine venines vera vet vetted vetter vetting we wed wedded wedder wee weed weeded weeder weedy ween weened weening weep weeper weepy weer wees weet weeted weeting weever weewee weeweed weewees wen were wet wetted wetter wetting xi xylol xylose xyloses xylyl zill zin zine zines zing zit ziti zizit zzz

Even though our program allows the arrangement to spread out quite a bit during the search, the best arrangements we found tend to be densely packed. This is not unexpected: densely-packed arrangements lead to a larger number of different letter pairs connected by knight's moves, which in turn usually leads to more words. Our best solution shown above fits in a 9x8 rectangle; since the standard chessboard is 8x8 it is interesting to seek the best arrangement for that size. The best we found scores just one point less (1477 words):

H		P		N		X	
	W	G	M	Y	V		
	O		E		I		
	R		L		D		Z
C		S		T		F	
		A		U			
J	K						
	Q		B				

This was obtained from the previous solution by removing the left column and relocating the J to be knight-adjacent to the vowel A instead of the O. This causes the 12 JO- words in the above list to be replaced by these 11 JA words:

ajar jab jacal jack jacks jalop jaloppy jalopy jar jarred raja

We think that both our 9x8 and 8x8 arrangements are pretty good but we really have no idea how close to the truly optimal solutions they are, so the reader is challenged to try improving on them. Although hill-climbing is often surprisingly effective, perhaps a completely different strategy would work better.

What about even smaller grids? Our best 7x7 was obtained by lopping off the right column and bottom row from the 8x8, relocating the B,Q,Z at random, then hill-climbing from there. It has 1446 words.

H		P		N		F
	D	G	M	Y	V	
	O		E		I	X
	R	U	L		W	
C		S		T		Z
	J	A				
Q	K			B		

For smaller square grids (6x6 and 5x5) we used random starting arrangements with E placed near the center followed by hill climbing. In the 5x5 grid there are no unoccupied squares and one letter of the alphabet has to be left out. Here are our best 6x6 and 5x5 grids:

	J	I	Z		
M	Y	F		N	K
	T	O	L	Q	V
B	U	X	E	C	G
	D	A	H		W
P		S		R	

1377 words

J	P	Z	T	H
D	I	C	Y	V
O	X	E	M	A
N	U	L	K	W
F	S	G	R	B

1155 words

Puzzle #2: Longest Knight-Graphable String in a Published Work

A well-known word recreation is to scan published texts for strings of letters that possess a certain logological property, such as being heteroliteral text, a pangram, or a pi mnemonic. So, in this vein, we can look for knight-graphable strings of text (or *knight's-move text*) in published works.

In a 1998 *Word Ways* article [2], Ross Eckler discusses the queen's-move variant of this question, and reports that the first 39 letters of the Gettysburg Address ("Fourscore and seven years ago our fathers brou") can be spelled out by a chess queen on a suitably labelled board. (Note that in queen's-move text the squares passed over by the queen on each move must be empty.) We think it is a new result that this can be improved to 40 letters (up to "broug") with the board shown below. We also determined by exhaustive search that 41 letters is impossible.

					H
		V	D		
	B	S	E	N	
		R	A	Y	
C	O		F		
	G	U			T

Knight's-move text fares much less well in Gettysburg, with only the first six letters (FOURSC) being traversable by a chess knight. Adding one more letter to give FOURSCO causes the underlying graph to have a 5-cycle (OURSCO) and, as we discussed in our previous article on NG words, an odd-length cycle makes knight-graphing impossible.

To find longer examples of knight's-move text we searched the entire contents of the 2010 Project Gutenberg DVD, which contains about 30,000 books.. For every starting position in each text we tested every window length up to the first non-NG length. This is quite computationally intensive, because each of the typically 100,000+ trial strings in a single book involves an exhaustive search of all possible board labellings until we either find one that works or determine that there is none. However, with some optimizations we were able to search all 30,000 books in a reasonable amount of time.

As often happens when looking for long strings of running text that satisfy a certain constraint, all of the longest examples of knight's-move text that we found have a fair amount, and sometimes an inordinant amount, of repetition. We rejected outright any strings with severe repetition but decided to allow some repeated words or phrases as long as they satisfied our vague aesthetic notion of acceptable. The longest knight-graphable text that we found (allowing some, but not too much, word repetition) has 50 letters, and is from Thomas Huxley's *Collected Essays* (published 1893-94), volume 3 (*Science and Education*), section 10 (*On The Study Of Biology*):

"...they can name [**the muscles of the dog by the names of the muscles of the man, an**]d the nerves of the dog by those of the nerves of the man..."

					Y	
		F		G		
			M	T	D	B
	U	L	O		A	H
C				E		
		S				N

We consider the next example, from Emanuel Swedenborg's *The Delights of Wisdom Pertaining to Conjugial Love*, to be one of the best. It is 46 letters long and includes 22 different letters of the alphabet (only K,Q,X,Z are missing). It does have one repeated word ("the"), however.

"...those who do not love their wives turn themselves away from them, and in the day-time regar[**d them with aversion. By the reception of the conjugial s**]phere by the husband..."

	G	C				
W	V			O	P	
U		I	E			F
A	M		N	T	R	
	J	H	S		B	D
	L		Y			

The longest knight's-move string we found with no repeated words is from Robert Browning's poem *The Pied Piper of Hamelin*. It is 44 letters long, contains 20 different letters of the alphabet and fits on a tiny 6x5 board. It is also nice that it starts and ends on a word boundary and ends on a line boundary.

"Rouse up, sirs! Give **your brains a racking To find the remedy we're lacking**."

	K	D	U	M	
Y	R		I	T	
C	N	O	E		
B	W	A	G	F	H
S				L	

References

[1] Tristan Miller and Mike Keith, "Knight-Graphable Words", *Word Ways*, 48(1), February 2015.

[2] A. Ross Eckler, "Queen's-Move Text", *Word Ways*, 31(3), August 1998.